International Workshop on Differential and Difference Equations, q-Series, and Related Problems

May 4-6, 2025

微分与差分方程、q-级数及相关问题研讨会

程序册

Program

大湾区大学，Great Bay University
Songshan Lake Campus, Dongguan, China
广东省东莞市松山湖园区大学路 16 号

The International Workshop on Differential and Difference Equations, q-Series, and Related Problems aims to bring together experts, scholars, and researchers from both domestic and international communities to discuss the latest research advancements in differential and difference equations, q-difference equations, q-series, and related fields. We welcome interested scholars, researchers, and graduate students to participate.

This workshop offers a unique opportunity for participants to share ideas and present their work with leading experts in the field. We look forward to your participation.

Organizers:

Guoting Chen, guoting.chen@gbu.edu.cn, Great Bay University,

Changgui Zhang, changgui.zhang@univ-lille.fr, University of Lille, France,

Confirmed Speakers:

Qingdan DU, Hebei Normal University

Bing HE, Central South University

Zhiguo LIU, East China Normal University

Jean-Pierre RAMIS, University of Toulouse, France

Jacques SAULOY, University of Toulouse, France

Kouichi TAKEMURA, Ochanomizu University, Japon

Yilei TANG, Shanghai Jiaotong University

Boxue WANG, Wuhan University

Xiaomeng XU, Peking University

Wentao ZENG, Wuhan University

Changgui ZHANG, University of Lille, France

Huohong ZHANG, Wuhan University

Nianhong ZHOU, Guangxi Normal University

Yue ZHOU, Central South University

Confirmed Attendees (to be completed):

Jiayu CAI, Harbin Institute of Technology, Shenzhen

Guoting CHEN, Great Bay University

Huan DAI, Harbin Institute of Technology, Shenzhen

Yingyi FU, Harbin Institute of Technology, Shenzhen

Cheng HE, Sun Yat-sen University and Great Bay University

Zipo JIBAO, Harbin Institute of Technology, Shenzhen

Wenqiao LI, Academy of Mathematics and System Sciences, CAS

Jinghong LIN, University of Chinese Academy of Sciences

Sikai LIN, Harbin Institute of Technology, Shenzhen

Changjian LIU, Sun Yat-sen University (Zhuhai)

Yiming MA, Peking University

Changsong SHI, Wuhan University

Mengfeng SUN, Shanghai University

Fudong WANG, Great Bay University

Caius WOJCIK, Harbin Institute of Technology, Shenzhen

Guanyuan ZENG, Harbin Institute of Technology, Shenzhen

Xiaojie ZHANG, Guangzhou University

Yulin ZHAO, Sun Yat-sen University (Zhuhai)

Local organization:

Yingyi FU, 23S058019@stu.hit.edu.cn, Tel. 183 8823 1546 (微信)

Conference hotel:

Dong Guan Yingbin Residence, Contact 迎宾里酒店彭经理 17307896119

450 yuans/night (conference rate).

Registration:

May 4, From 15:00 at Dong Guan Yingbin Residence

Program

Monday, May 5, 2025, room B302

Chairman:		
8:30-9:15	Kouichi TAKEMURA	On q-Heun equation
9:15-10:00	Zhiguo LIU	A new q-partial differential equation
10:00-10:15	Tea Break	
10:15-11:00	Yilei TANG	Center problem and local integrability of generalized Kukles systems
11:00-11:45	Xiaomeng XU	Stokes phenomenon, isomonodromy deformation and representation theory
11:45-13:00	Lunch	

Chairman:		
14:00-14:45	Jacques SAULOY	Mano decomposition of q-monodromy matrices (Zoom 925 0504 6483, password: 275009)
14:45-15:30	Bing HE	Some expansion formulas for q-series and their applications
15:30-15:45	Tea Break	
15:45-16:30	Jean-Pierre RAMIS	Painlevé VI and q-Painlevé VI, Analogies and Relations
16:30-17:15	Yue ZHOU	The constant term conjectures
18:00-20:00	Dinner	

Tuesday, May 6, 2025, room B302

Chairman:		
8:30-9:15	Qingdan DU	Algorithmic approaches on identities of partition functions
9:15-10:00	Boxue WANG	Mizuno's rank three Nahm sums I: identities of index $(1, 1, 2)$
10:00-10:15	Tea Break	
10:15-11:00	Huohong ZHANG	Modularity of some Nahm sums as vector-valued functions
11:00-11:45	Nianhong ZHOU	Monotonicity of Ranks in Concave Compositions
11:45-13:00	Lunch	

Chairman:		
14:00-14:45	Wentao ZENG	Rogers–Ramanujan type identities for rank two partial Nahm sums
14:45-15:30	Changgui ZHANG	Analytical approach to Ramanujan's ${}_1\psi_1$-summation formula via q-difference equations
15:30-15:45	Tea Break	

Titles and abstracts of the talks

Qingdan DU, Algorithmic approaches on identities of partition functions.

Abstract. Ramanujan discovered three celebrated congruences for the partition function and some identities on the partition function. In this talk, we will present algorithms to compute the Ramanujan-type identities and on vanishing coefficients with arithmetic progressions in the sum of two generalized eta-quotients by using the theory of modular forms. This talk is based on joint works with William Y.C. Chen and Jack C.D. Zhao, and D. Tang.

Bing HE, Some expansion formulas for q-series and their applications.

Abstract. In this talk, I discuss some general expansion formulas for q-series, which were motivated by three of Liu's identities. These expansion formulas include as special cases or limiting cases many q-identities including the q-Gauss summation formula, the q-Pfaff-Saalschütz summation formula, three of Jackson's transformation formulas and Sears'terminating 4 3 transformation formula. Some applications are also presented.

Zhiguo LIU, A new q-partial differential equation.

Abstract. A q-partial differential equation is a q-analogue of a partial differential equation. A few q-partial differential equations have been studied in the past decade and q-partial differential equations are an important tool for deriving q-series identities. In this talk, I will introduce a new q-partial differential equation and discuss its application in q-series.

Jean-Pierre RAMIS, Painlevé VI and q-Painlevé VI, Analogies and Relations.

Abstract. We have today a very complete knowledge of P_{VI}, the sixth Painlevé equation discovered around 1905 by Painlevé and Gambier on one side and R. Fuchs on another. But the situation is very different for q-P_{VI}, its q-analog discovered by Jimbo and Sakai in 1996 using a Lax pair.

I will give a parallel presentation of the two cases, limiting myself to (a part of) what is known today for q-P_{VI} and to the generic cases. I will insist on the right side of the Riemann-Hilbert and Riemann-Hilbert-Birkhoff correspondences.

I will not speak of the symmetries of the equations, of the delicate problems of moduli of connections in the non-generic cases and of the Galoisian aspects.

A part of my lecture is a review of classical results. Others come from joint works with Yousuke Ohyama and Jacques Sauloy and from recent results of Nalini Joshi, Marta Mazzocco and Pieter Roffelsen.

Jacques SAULOY, Mano decomposition of q-monodromy matrices.

Abstract. Based on Birkhoff's idea of rational classification of rational q-difference systems through their so-called "connection matrix", it is possible to define a good q-analogue of monodromy encoded by an invertible meromorphic matrix; this, without any use of multivaluedness but with a crucial meaning of the nature of its singularities. However, contrary to the case of differential equations, it has proved difficult to isolate the local contributions of those singularities. Yet, in the case of the Jimbo-Sakai family (which was introduced as a Lax-pair form of q-PVI), where four singularities appear, a very useful tool, the Mano decomposition, provides means to localize at pairs of singularities. I shall describe Mano decomposition. Some of its geometrical consequences will be displayed in Ramis' talk. (All this comes from a joint work with Yousuke Ohyama and Jean-Pierre Ramis.)

Kouichi TAKEMURA, On q-Heun equation.

Abstract. Heun's differential equation is a standard form of the second order Fuchsian differential equation with four singularities $\{0, 1, t, \infty\}$, and it is written as

$$\frac{d^2y}{dx^2} + \Big(\frac{\gamma}{x} + \frac{\delta}{x-1} + \frac{\epsilon}{x-t}\Big)\frac{dy}{dx} + \frac{\alpha\beta x - B}{x(x-1)(x-t)}y = 0,$$

under the condition $\gamma + \delta + \epsilon = \alpha + \beta + 1$. The q-Heun equation is a q-difference equation written as

$$\{a_2x^2 + a_1x + a_0\}g(x/q) - \{b_2x^2 + b_1x + b_0\}g(x) + \{c_2x^2 + c_1x + c_0\}g(xq) = 0,$$

with the condition $a_2a_0c_2c_0 \neq 0$, and it tends to Heun's differential equation as $q \to 1$. It was introduced by Hahn (1971) and was rediscovered around 2016 by considering degeneration of the Ruijsenaars-van Diejen system four times.

In the talk, I will review some properties of the q-Heun equation. Some special solutions will be discussed.

Yilei TANG, Center problem and local integrability of generalized Kukles systems.

Abstract. In this talk we investigate the center problem and local integrability for generalized Kukles systems. We derive sufficient and necessary conditions for the origin of such systems with $\mathbb{Z}_2$-symmetry to be a center. Moreover, we provide examples to illustrate the center conditions using our theoretical results and give a negative answer to a conjecture proposed in the literature.

Boxue WANG, Mizuno's rank three Nahm sums I: identities of index $(1, 1, 2)$.

Abstract. Mizuno provided 19 examples of generalized rank three Nahm sums with sym-

metrizer $D = \mathrm{diag}(1, 1, 2)$ which are conjecturally modular. We confirm their modularity by establishing Rogers-Ramanujan type identities of index $(1, 1, 2)$ for these examples. We first reduce these Nahm sums to some double sums or single sums, and then we use some known identities or apply the theory of Bailey pairs to prove the desired identities. Meanwhile, we generalize some triple sum identities to general multi-sum identities.

Xiaomeng XU, Stokes phenomenon, isomonodromy deformation and representation theory.

Abstract. This talk will unveil some relations between the meromorphic linear system of differential equations and the representation theory. In particular, it shows that the Stokes phenomenon at a second order pole is characterized by quantum groups, while the WKB approximation is encoded by crystal structures. The latter result relies on an explicit formula relating the Stokes matrices to the long time asymptotics of the isomonodromy deformation equations of the linear system. If time allowed, it will talk about some generalizations of the results to the q-difference equations setting.

Wentao ZENG, Rogers–Ramanujan Type Identities for Rank Two Partial Nahm Sums.

Abstract. Let A be a $r \times r$ rational nonzero symmetric matrix, B a rational column vector, C a rational scalar. For any integer lattice L and vector v of Z^r, we define Nahm sum on the lattice coset $v + L \in Z^r/L$:

$$f_{A,B,C,v+L}(q) := \sum_{n=(n1,\ldots,nr)T \in v+L} \frac{q^{\frac{1}{2}n^T An + n^T B + C}}{(q;q)_{n_1} \cdots (q;q)_{n_r}}.$$

If L is a full-rank lattice and a proper subset of Z^r, then we call $f_{A,B,C,v+L}(q)$ a rank r partial Nahm sum. When the rank $r = 1$, we find eight modular partial Nahm sums using some known identities. When the rank $r = 2$ and L is one of the lattices $Z(2, 0) + Z(0, 1)$, $Z(1, 0) + Z(0, 2)$ or $Z(2, 0) + Z(0, 2)$, we find 14 types of symmetric matrices A such that there exist vectors B, v and scalars C so that the partial Nahm sum $f_{A,B,C,v+L}(q)$ is modular. We establish Rogers–Ramanujan type identities for the corresponding partial Nahm sums which prove their modularity.

Changgui ZHANG, Analytical Approach to Ramanujan's ${}_1\psi_1$-Summation Formula via q-Difference Equations.

Abstract. Each bilateral basic series ${}_m\psi_m(a_1, ..., a_m; b_1, ..., b_m; q, z)$ satisfies an m-th order Fuchsian linear q-difference equation $\Delta y(z) = 0$, where a_j, $b_j \in \mathbf{C}^*$,

$$\Delta = \Delta^{a_1,\ldots,a_m}_{b_1,\ldots,b_m} = \prod_{j=1}^{m} \left(1 - \frac{b_j}{q}\sigma_q\right) - z \prod_{j=1}^{m} (1 - a_j\sigma_q),$$

with $\sigma_q y(z) = y(qz)$. The Ramanujan's ${}_1\psi_1$-summation formula states that

$$ {}_1\psi_1(a;b;q,z) = \frac{(q,b/a,az,q/az;q)_\infty}{(b,q/a,z,b/az;q)_\infty}.$$

By noticing that $\dfrac{(az,q/az;q)_\infty}{(z,b/az;q)_\infty}$ satisfies the same functional equation as ${}_1\psi_1(a;b;q,z)$, there exists a meromorphic function $C(a,b;q,z)$ in $\mathbf{C}^*$ such that

$$ {}_1\psi_1(a;b;q,z) = C(a,b;q,z)\frac{(az,q/az;q)_\infty}{(z,b/az;q)_\infty}, \quad C(a,b;q,qz) = C(a,b;q,z).$$

Thus, it suffices to explain why $C(a,b;q,z)$ reduces to the constant function $\dfrac{(q,b/a;q)_\infty}{(b,q/a;q)_\infty}$. For doing that, we will compute $C(a,b;q,q)$ using the Jackson integral, employing a contour integral with the aid of the q-logarithm

$$ \ell(q;z) = \frac{\delta\theta(-z)}{\theta(-z)} = -z\frac{\theta'(-z)}{\theta(-z)},$$

where $\theta(z) = \sum_{n\in\mathbb{Z}} q^{n(n-1)/2}z^n$.

Furthermore, we will explain why this approach can be generalized to derive connection formulas for ${}_m\psi_m(a_1,...,a_m;b_1,...,b_m;q,z)$ within a broader framework.

Huohong ZHANG, Modularity of Some Nahm Sums as Vector-valued Functions.

Abstract. Zagier observed that modular Nahm sums associated with the same matrix may form a vector-valued modular function on some congruence subgroup. We establish modular transformation formulas for several families of Nahm sums by viewing them as vector-valued functions, and thereby we show that they are indeed modular on the congruence subgroup $\Gamma_0(N)$ with $N = 1, 2, 3, 4$. In particular, we prove two transformation formulas discovered by Mizuno related to the Kanade–Russell mod 9 conjecture and Capparelli's identities. We also establish vector-valued transformation formulas for some theta series. As applications, we give modular transformation formulas for various families of Nahm sums involving those in the Andrews–Gordon identities and Bressoud's identities. This talk is based on a joint work with Liuquan Wang.

Nianhong ZHOU, Monotonicity of Ranks in Concave Compositions

Abstract. A (strongly) concave composition of an integer n is a sequence of positive integers that is (strictly) decreasing to a point and then (strictly) increasing thereafter, such that the sum of the entries equals n. The difference between the number of entries before and after the low point of the sequence is referred to as the rank of the (strongly) concave composition. By establishing the q-difference systems that the rank generating

functions satisfy, we prove monotonicity for the ranks of strongly concave compositions and concave compositions for each positive integer n.

Yue ZHOU, The constant term conjectures.

Abstract. The study of constant term identities is a branch of combinatorics. The research on constant term identities originated from a conjecture by Freeman Dyson, a Wolf Prize winner in 1962. In this talk, we will review the important conjectures in this field. Meanwhile, we will present our recent progress on the 26-year-old q-Baker-Forrester conjecture.